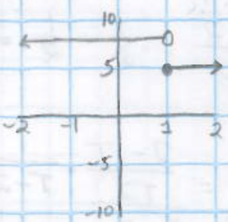
2.2 Homework Problems

See bottom of page 3 for comments about this sample.

10. If $f(1)=5$, must $\lim_{x\to 1} f(x)$ exist?

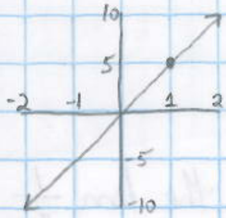
This problem is stating that the point (1,5) exists on the graph of the unknown function f(x). It then asks if the limit of f(x) exists at the point (1,5). Just because the point (1,5) exists does <u>not</u> mean the limit exists at this point. The limit only exists if the function approaches the same exact point from both sides of $x=1$. Because we do not know the equation of f(x), the fact that the limit exists as f(x) approaches $x=1$, cannot be assumed. This also means we cannot conclude anything about the limit at $x=1$.

If the limit does not exist at the point (1,5), the function may look like a unit step function:



f(x) is not approaching the same point from both sides of $x=1$.

However, if the limit does exist at the point (1,5), then $\lim_{x\to 1} f(x)=5$, because we know for a fact that $f(1)=5$. Therefore, when the limit exists, f(x) will approach $y=5$ from both sides of $x=1$. An example graph of this concept would look like:



66. a) What can we infer about:

$$\lim_{x \to 0} \frac{1-\cos x}{x^2}$$

if the following inequalities hold true for values of x close to zero?:

$$\frac{1}{2} - \frac{x^2}{24} < \frac{1-\cos x}{x^2} < \frac{1}{2}$$

Using the Sandwich Theorem, we can conclude that $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

The Sandwich Theorem uses three functions and assumes that $g(x) \leq f(x) \leq h(x)$ over an open interval containing the value c. However, a point does not have to exist at $x=c$. Under these conditions the theorem states:

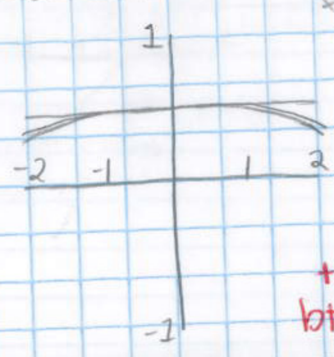
$$\text{If } \lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$$

$$\text{Then } \lim_{x \to c} f(x) = L$$

Now looking at the problem, $\frac{1}{2}$ can be substituted for $h(x)$ in the theorem and $\frac{1}{2} - \frac{x^2}{24}$ can be substituted for $g(x)$. The limit for both of these functions as $x \to 0$ is $\frac{1}{2}$. Therefore, using $\frac{1-\cos x}{x^2}$ as $f(x)$ in the Sandwich Theorem, we can conclude that $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$.

Why can you use Sandwich Thm?

b)



Looking at all three functions, the y-values get closer to $\frac{1}{2}$ as x approaches 0.

the graph f(x) also clearly lies btw g(x) and h(x).

2.2 Homework Problems

77. We know the following conditions:

$x^4 \le f(x) \le x^2$ for x in $[-1,1]$

$x^2 < f(x) < x^4$ for x in $x<1$ and $x>1$

According to the Sandwich Theorem,

If $\lim_{x\to c} x^4 = \lim_{x\to c} x^2 = L$

Then $\lim_{x\to c} f(x) = L$

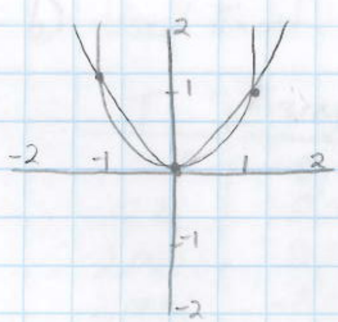
Using the conditions and theorem, I know the $\lim_{x\to c} f(x)$ for the x-values 0, 1, and -1.

$\lim_{x\to 0} x^4 = \lim_{x\to 0} x^2 = 0$ therefore $\lim_{x\to 0} f(x) = 0$

$\lim_{x\to 1} x^4 = \lim_{x\to 1} x^2 = 1$ therefore $\lim_{x\to 1} f(x) = 1$

$\lim_{x\to -1} x^4 = \lim_{x\to -1} x^2 = 1$ therefore $\lim_{x\to -1} f(x) = 1$

These conclusions are evident on the graph of the two functions together.



This is a student's writing assignment for 3 problems from the first assignment from Calc 1 in Fall 2011. Each question was graded on a 20 point scale (see the grading criteria elsewhere).

Because there are only a few minor problems, this assignment received a grade of 55/60.